Pre-AP Geometry 8-6 Study Guide: Vectors (pp 577-580)

<u>Attendance Problems</u>. Find the distance between the points.

1. (0, 15) & (17, 0) **2.** (-4, 2) & (4, -2)

Solve each equation. Round to the nearest tenth or nearest degree.

3. $\cos 26° = \frac{x}{7}$ **4.** $\sin 26° = \frac{y}{7}$ **5.** $\tan P = \frac{41}{53}$

6. $\tan Q = \frac{13}{11}$

- I can find the magnitude and direction of a vector.
- I can use vectors and vector addition to solve real world problems.

<u>Common Core</u>: CC.9-12.G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

<u>Vocabulary</u>

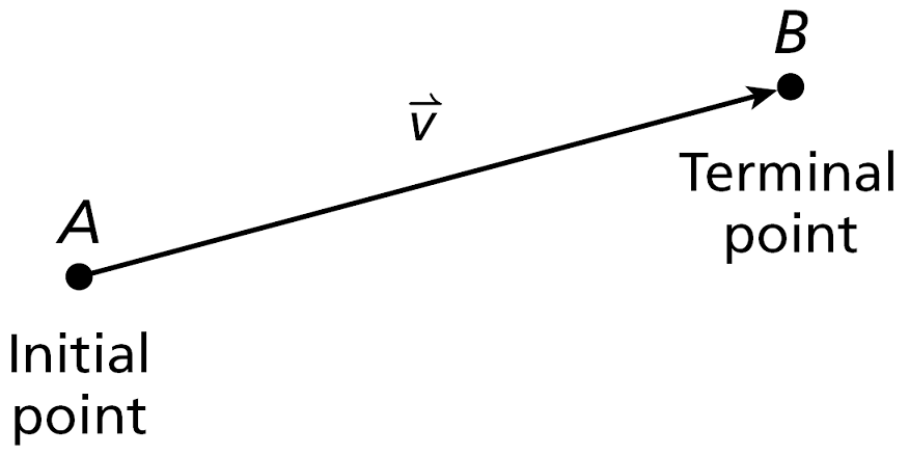
vector	component form	magnitude
direction	equal vectors	parallel vectors
resultant vector		

Q: Which trigonometric ratios do pig farmers use?
A: Swine (sine) and coswine (cosine)!

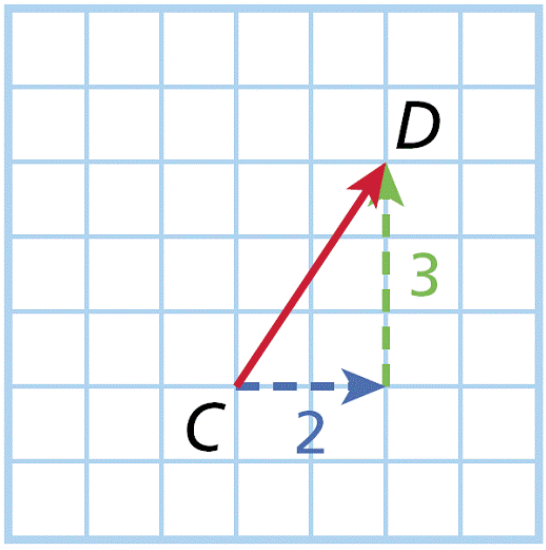
"Lean too much upon the approval of people, and it becomes a bed of thorns."—
19th Century Educator, Tehyi Hsieh

The speed and direction an object moves can be represented by a *vector*. A **vector** is a quantity that has both length and direction.

You can think of a vector as a directed line segment. The vector below may be named $\overrightarrow{AB}$ or $\vec{v}$.



A vector can also be named using *component form*. The **component form** $\langle x, y \rangle$ of a vector lists the **horizontal** and **vertical** change from the initial point to the terminal point. The component form of $\overrightarrow{CD}$ is <**2**, **3**>.



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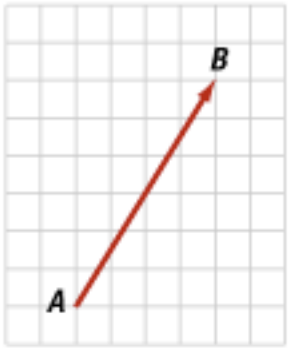
Video Example 1. Write the vector in component form.

A.



B.

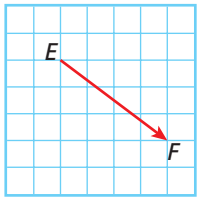
$\overrightarrow{PQ}$ with $P(6, 3)$ and $Q(-4, 1)$

1 Writing Vectors in Component Form

Write each vector in component form.

A $\overrightarrow{EF}$

The horizontal change from E to F is 4 units.
The vertical change from E to F is -3 units.
So the component form of $\overrightarrow{EF}$ is $\langle 4, -3 \rangle$.



B $\overrightarrow{PQ}$ **with** $P(7, -5)$ **and** $Q(4, 3)$

$\overrightarrow{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$ *Subtract the coordinates of the initial point from the coordinates of the terminal point.*

$\overrightarrow{PQ} = \langle 4 - 7, 3 - (-5) \rangle$ *Substitute the coordinates of the given points*

$\overrightarrow{PQ} = \langle -3, 8 \rangle$ *Simplify.*

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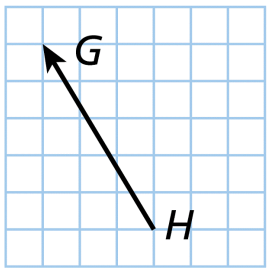
Example 1. Write the vector in component form.

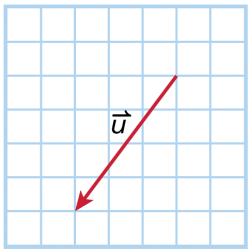
A.



B. $\overline{MN}$ with M(-8, 1) and N(2, -7)

Guided Practice. Write the vector in component form.

7. $\vec{u}$



8. The vector with initial point $L(-1, 1)$ and terminal point $M(6, 2)$.

The **magnitude** of a vector is its length. The magnitude of a vector is written $|\overline{AB}|, \|\overline{AB}\|, or\ |\vec{v}|$.

When a vector is used to represent speed in a given direction, the magnitude of the vector equals the speed. For example, if a vector represents the course a kayaker paddles, the magnitude of the vector is the kayaker's speed.

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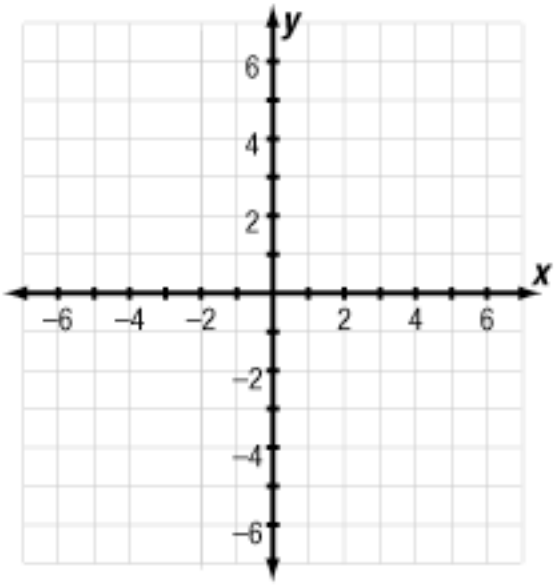
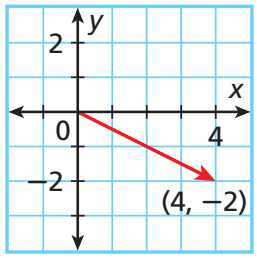
Video Example 2.

Draw the vector ⟨–2, 5⟩ on a coordinate plane. Find its magnitude to the nearest tenth.



2 Finding the Magnitude of a Vector

Draw the vector ⟨4, –2⟩ on a coordinate plane. Find its magnitude to the nearest tenth.



Step 1 Draw the vector on a coordinate plane. Use the origin as the initial point. Then $(4, -2)$ is the terminal point.

Step 2 Find the magnitude. Use the Distance Formula.

$$\left|\langle 4, -2\rangle\right| = \sqrt{(4-0)^2 + (-2-0)^2} = \sqrt{20} \approx 4.5$$

Example 2. Draw the vector <–1, 5> on a coordinate plane. Find its magnitude to the nearest tenth.

9. Guided Practice. Draw the vector <–3, 1> on a coordinate plane. Find its magnitude to the nearest tenth.

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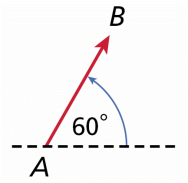
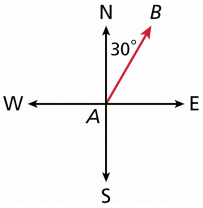
The **direction** of a vector is the angle that it makes with a horizontal line. This angle is measured counterclockwise from the positive *x*-axis. The direction of $\overline{AB}$ is 60°.

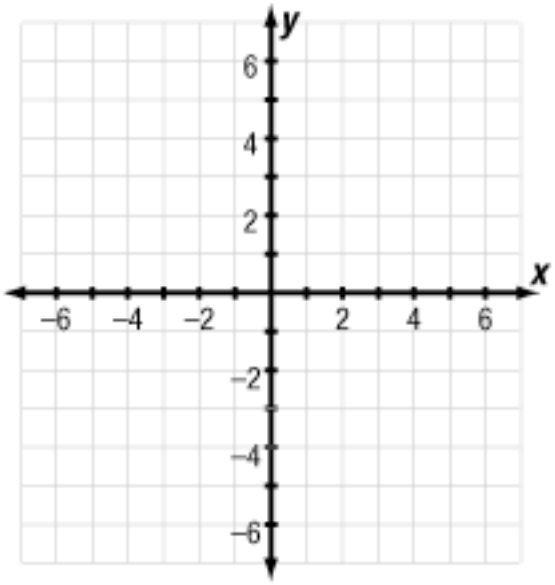


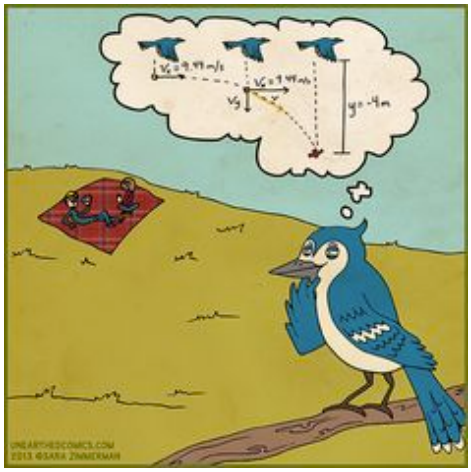
The direction of a vector can also be given as a bearing relative to the compass directions *north*, *south*, *east*, and *west*. $\overline{AB}$ has a bearing of N 30° E.



Video Example 3.

A wind velocity is given by the vector ⟨3, 5⟩. Draw the vector on a coordinate plane. Find the direction of the vector to the nearest degree.





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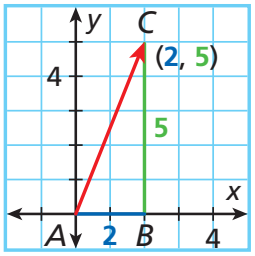
3 Finding the Direction of a Vector

A wind velocity is given by the vector ⟨2, 5⟩.
Draw the vector on a coordinate plane.
Find the direction of the vector to the nearest degree.

Step 1 Draw the vector on a coordinate plane.
Use the origin as the initial point.



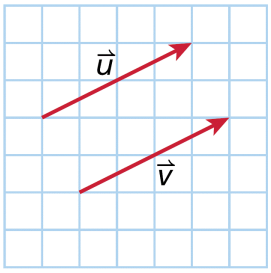
Step 2 Find the direction.
Draw right triangle ABC as shown. $\angle A$ is the angle formed by the vector and the x-axis, and $\tan A = \frac{5}{2}$. So $m\angle A = \tan^{-1}\left(\frac{5}{2}\right) \approx 68°$.

Example 3. The force exerted by a skier is given by the vector <1, 4>. Draw the vector on a coordinate plane. Find the direction of the vector to the nearest degree.

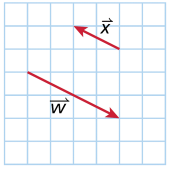
10. Guided Practice. The force exerted by a tugboat is given by the vector <7, 3>. Draw the vector on a coordinate plane. Find the direction of the vector to the nearest degree.

8-6 Vectors (pp 582) 19-21, 23, 25-26.

Two vectors are **equal vectors** if they have the same magnitude and the same direction. For example, $\vec{u} = \vec{v}$. Equal vectors do not have to have the same initial point and terminal point.



Two vectors are **parallel vectors** if they have the same direction or if they have opposite directions. They may have different magnitudes. For example, $\vec{w} \parallel \vec{x}$. Equal vectors are always parallel vectors.



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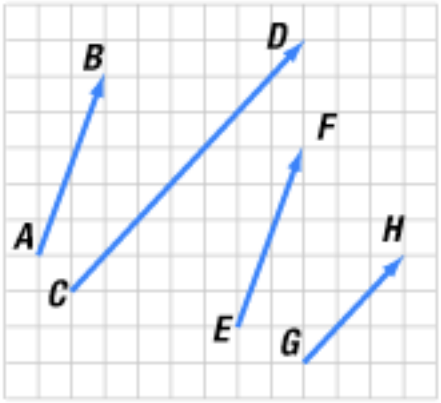
Example 4.

Identify each of the following.

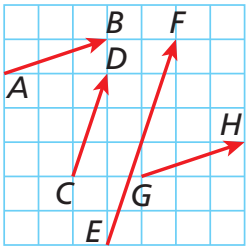


equal vectors — *Identify vectors with the same magnitude and direction.*

parallel vectors — *Identify vectors with the same or opposite directions.*

4 Identifying Equal and Parallel Vectors

Identify each of the following.



A equal vectors

$\overrightarrow{AB} = \overrightarrow{GH}$ *Identify vectors with the same magnitude and direction.*

B parallel vectors

$\overrightarrow{AB} \parallel \overrightarrow{GH}$ and $\overrightarrow{CD} \parallel \overrightarrow{EF}$ *Identify vectors with the same or opposite directions.*

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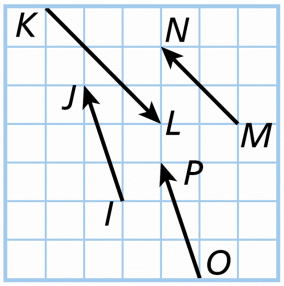
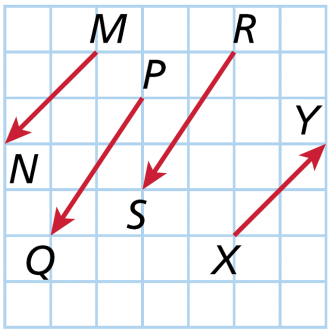
Example 4. Identify each of the following.
A. Equal vectors **B.** Parallel vectors



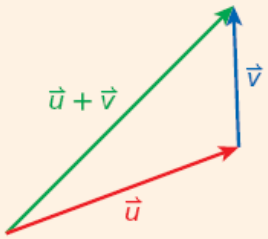
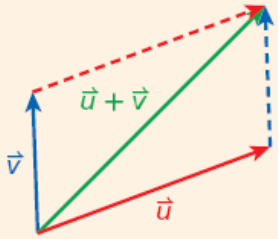
Guided Practice. Identify each of the following.
11. Equal vectors

12. Parallel vectors

The **resultant vector** is the vector that represents the sum of two given vectors. To add two vectors geometrically, you can use the head-to-tail method or the parallelogram method.



Vector Addition

METHOD	EXAMPLE
Head-to-Tail Method Place the initial point (tail) of the second vector on the terminal point (head) of the first vector. The resultant is the vector that joins the initial point of the first vector to the terminal point of the second vector.	
Parallelogram Method Use the same initial point for both of the given vectors. Create a parallelogram by adding a copy of each vector at the terminal point (head) of the other vector. The resultant vector is a diagonal of the parallelogram formed.	

To add vectors numerically, add their components. If $\overline{u} = \langle x_1, y_1 \rangle$ and $\overline{v} = \langle x_2, y_2 \rangle$, then $\overline{u}+\overline{v} = \langle x_1 + x_2, y_1 + y_2 \rangle$.

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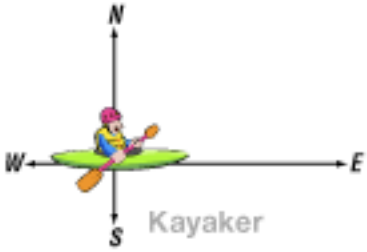
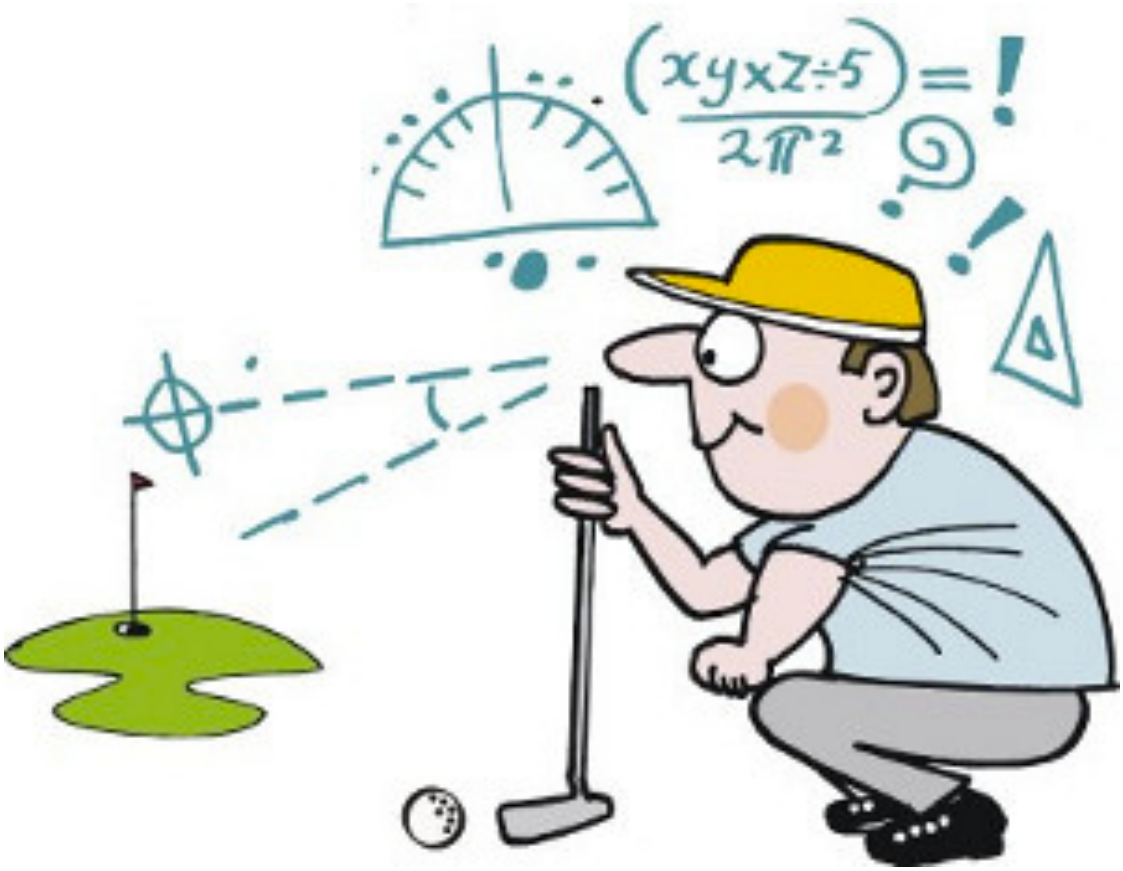
Video Example 5.

A kayaker leaves shore at a bearing of N 50° E and paddles at a constant speed of 5 mi/h. There is a 1 mi/h current moving due east. What are the kayak's actual speed and direction? Round the speed to the nearest tenth and direction to the nearest degree.





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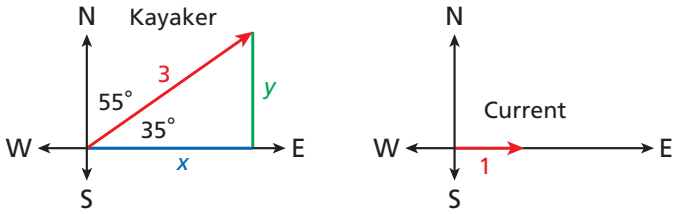
5 Sports Application

A kayaker leaves shore at a bearing of N 55° E and paddles at a constant speed of 3 mi/h. There is a 1 mi/h current moving due east. What are the kayak's actual speed and direction? Round the speed to the nearest tenth and the direction to the nearest degree.

Step 1 Sketch vectors for the kayaker and the current.



Step 2 Write the vector for the kayaker in component form.

The kayaker's vector has a magnitude of **3** mi/h and makes an angle of 35° with the x-axis.

$\cos 35° = \frac{x}{3}$, so $x = 3\cos 35° \approx 2.5$.

$\sin 35° = \frac{y}{3}$, so $y = 3\sin 35° \approx 1.7$.

The kayaker's vector is $\langle 2.5, 1.7 \rangle$.

Step 3 Write the vector for the current in component form.

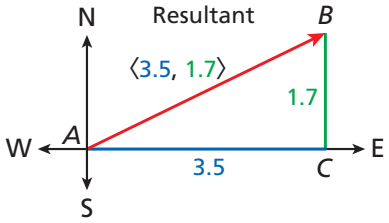
Since the current moves **1** mi/h in the direction of the x-axis, it has a horizontal component of **1** and a vertical component of **0.** So its vector is $\langle 1, 0 \rangle$.

Step 4 Find and sketch the resultant vector $\overrightarrow{AB}$.

Add the components of the kayaker's vector and the current's vector.

$\langle 2.5, 1.7 \rangle + \langle 1, 0 \rangle = \langle 3.5, 1.7 \rangle$

The resultant vector in component form is $\langle 3.5, 1.7 \rangle$.



Step 5 Find the magnitude and direction of the resultant vector.

The magnitude of the resultant vector is the kayak's actual speed.

$\left|\langle 3.5, 1.7 \rangle\right| = \sqrt{(3.5-0)^2 + (1.7-0)^2} \approx 3.9$ mi/h

The angle measure formed by the resultant vector gives the kayak's actual direction.

$\tan A = \frac{1.7}{3.5}$, so $A = \tan^{-1}\left(\frac{1.7}{3.5}\right) \approx 26°$, or N 64° E.

Example 5.
An airplane is flying at a constant speed of 400 mi/h at a bearing of N 70º E. A 60 mi/h wind is blowing due north. What are the plane's actual speed and direction? Round the speed to the nearest tenth and the direction to the nearest degree.

13. Guided Practice. Suppose the kayaker in the example instead paddles at 4 mi/h at a bearing of N 20° E. What are the kayak's actual speed and direction? Round the speed to the nearest tenth and the direction to the nearest degree.

8-6 Vectors
- (pp 582) 19-21, 23, 25-29, 31, 40, 49, 54, 58, 59.
- 8B Ready to Go On pretest & posttests.